FINANCIAL ECONOMETRICS

Master in Mathematical Finance

Master in Monetary and Financial Economics

Lecture 2 – Some Stylized Facts of Asset Returns

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Standard descriptive statistics

Let r_t, $t = 1,\ldots,N$ denote the log return of an asset at time *t*. The standard descriptive statistics are computed as

- Mean

$$\bar{r} = \frac{1}{n}\sum_{t=1}^{n} r_t$$

- Variance

$$s^2 = \frac{1}{n-1}\sum_{t=1}^{n}(r_t - \bar{r})^2$$

- Skewness

$$sk = \frac{1}{(n-1)}\sum_{t=1}^{n}\frac{(r_t - \bar{r})^3}{s^3}$$

- Kurtosis

$$k = \frac{1}{(n-1)}\sum_{t=1}^{n}\frac{(r_t - \bar{r})^4}{s^4}$$

Jarque-Bera test for normality

Under the normality assumption, *sk* and *k*-3 are distributed asymptotically as normal with zero mean and variance 6/*n* and 24/*n*, respectively.

- $H_0 : sk(r) = 0$ versus $H_0 : sk(r) \neq 0$. The test statistic is

$$\left| t = \frac{sk(r)}{\sqrt{6/n}} \right| > Z_{\alpha/2}$$

- $H_0 : k(r) = 3$ versus $H_0 : k(r) \neq 3$. The test statistic is

$$\left| t = \frac{k(r) - 3}{\sqrt{24/n}} \right| > Z_{\alpha/2}$$

- Combining the two tests (Jarque e Bera, 1987). The test statistic is

$$JB = \frac{sk^2(r)}{6/n} + \frac{[k(r) - 3]^2}{24/n} \sim \chi^2_{(2)}$$

	Stocks			
	IBM	KO	MSFT	PG
Mean	0.000311	0.000400	0.000820	0.000521
Median	0.000000	0.000000	0.000000	0.000000
Maximum	0.123585	0.093559	0.178822	0.090798
Minimum	-0.168854	-0.110669	-0.169769	-0.359942
Std. Dev.	0.019607	0.015697	0.022159	0.016118
Skewness	0.000687	-0.082119	-0.041307	-2.822514
Kurtosis	9.752590	7.037532	7.471314	66.49734
Jarque-Bera	7787.668	2788.795	3415.743	694059.5
Probability	0.000000	0.000000	0.000000	0.000000
Observations	4099	4099	4099	4099

Summary statistics for **daily log returns** for IBM, Coca-Cola, Microsoft and Procter & Gamble stocks. Sample period: Jun 11, 1990 – Set 12, 2006

	Stock Indexes				
	POR	UK	US	JAP	MAL
Mean	0.000157	0.000138	0.000230	-0.000125	8.10E-05
Median	0.000000	0.000194	0.000352	0.000000	0.000000
Maximum	0.097020	0.092649	0.110426	0.130618	0.232628
Minimum	-0.107755	-0.091580	-0.095137	-0.104351	-0.241591
Std. Dev.	0.010998	0.011958	0.012594	0.013893	0.015696
Skewness	-0.281274	-0.175207	-0.216154	-0.138284	0.785574
Kurtosis	11.85595	9.698833	11.52118	8.965752	47.33427
Jarque-Bera	12838.59	7336.411	11869.00	5815.156	320865.3
Probability	0.000000	0.000000	0.000000	0.000000	0.000000
Observations	3913	3913	3913	3913	3913

Summary statistics for **daily log returns** for Portugal, US, UK, Japan and Malaysia stock indexes. Sample period: Jan 2, 1995 – Dec 31, 2009

	Stock Indexes				
	POR	UK	US	JAP	MAL
Mean	0.001741	0.001389	0.002124	-0.001017	0.001026
Median	0.003389	0.003956	0.004942	5.53E-05	0.002344
Maximum	0.064767	0.036872	0.046990	0.055361	0.142659
Minimum	-0.096762	-0.060722	-0.082217	-0.102705	-0.135369
Std. Dev.	0.025463	0.018117	0.020530	0.023417	0.035493
Skewness	-0.661751	-0.864924	-0.878286	-0.477370	-0.070433
Kurtosis	4.977685	4.071640	4.489380	4.407777	6.048779
Jarque-Bera	42.23576	30.88335	39.55748	21.57969	69.47359
Probability	0.000000	0.000000	0.000000	0.000021	0.000000
Observations	179	179	179	179	179

Summary statistics for **monthly log returns** for Portugal, US, UK, Japan and Malaysia stock indexes. Sample period: Jan 2, 1995 – Dec 31, 2009

Example:
Calculate annualized returns and volatilities for daily log returns of POR, JAP, UK, US and MAL stock indexes.

Market	n	mean	sd	skew	kurt	mean×100	sd×100
JAP	3686	-0.00013	0.0143	-0,133	8,45	-0.0132	1.4314
POR	3770	0.00016	0.0112	-0.278	11.42	0.0163	1.1205
UK	3792	0.00014	0.0121	-0.174	9.40	0.0142	1.2148
US	3778	0.00024	0.0128	-0.214	11.13	0.0238	1.2817
MAL	3697	0.00009	0.0161	0.763	44.72	0.0086	1.6148

Some empirical properties of returns

- **Equity risk premia**
  "The expected return from an investment in a market of stocks exceeds the return from riskless investments…" (Taylor, 2005). Dimson, Marsh e Staunton (2002) provided information about the equity risk premium for many countries during the twentieth century. The difference between the annualized returns for stocks and bills from 1900 to 2000 is about 5.3%.

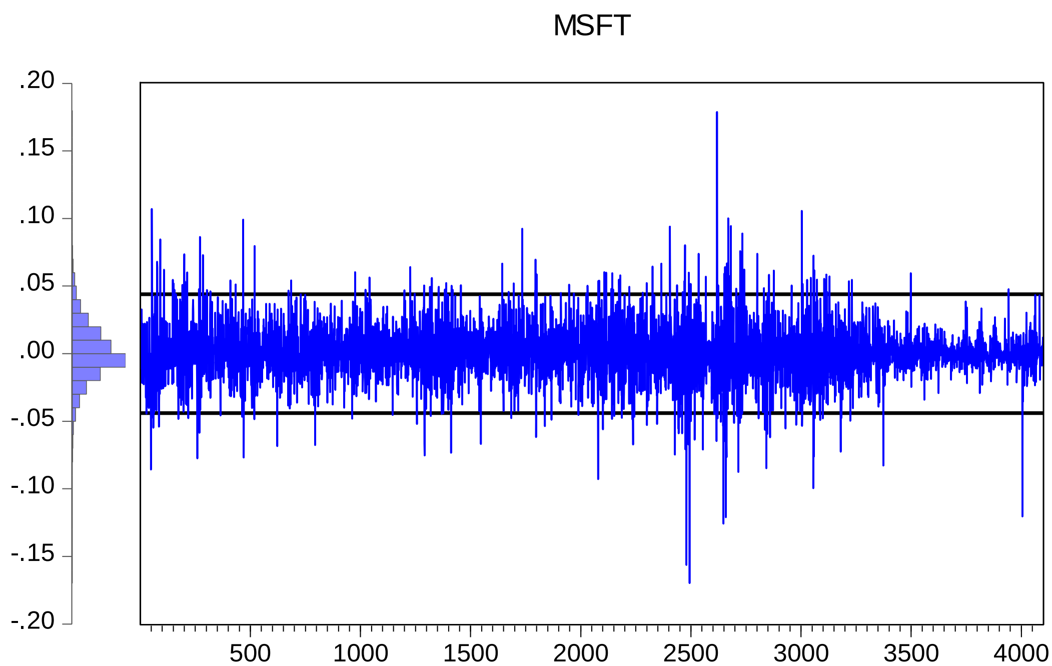
- **Standard deviations**
  Returns from currencies and bonds have lower variability than equity indices.
  Many empirical studies have argued that unconditional standard deviations increase as the firm size decreases.
  Monthly returns tend to have higher standard deviations than daily returns.

- **Skewness and kurtosis**
  These statistics are sensitive to extreme observations.
  In general, returns for stocks and equity indices exhibit negative skewness and are leptokurtic.
  Daily returns tend to have higher excess of kurtosis than monthly returns.

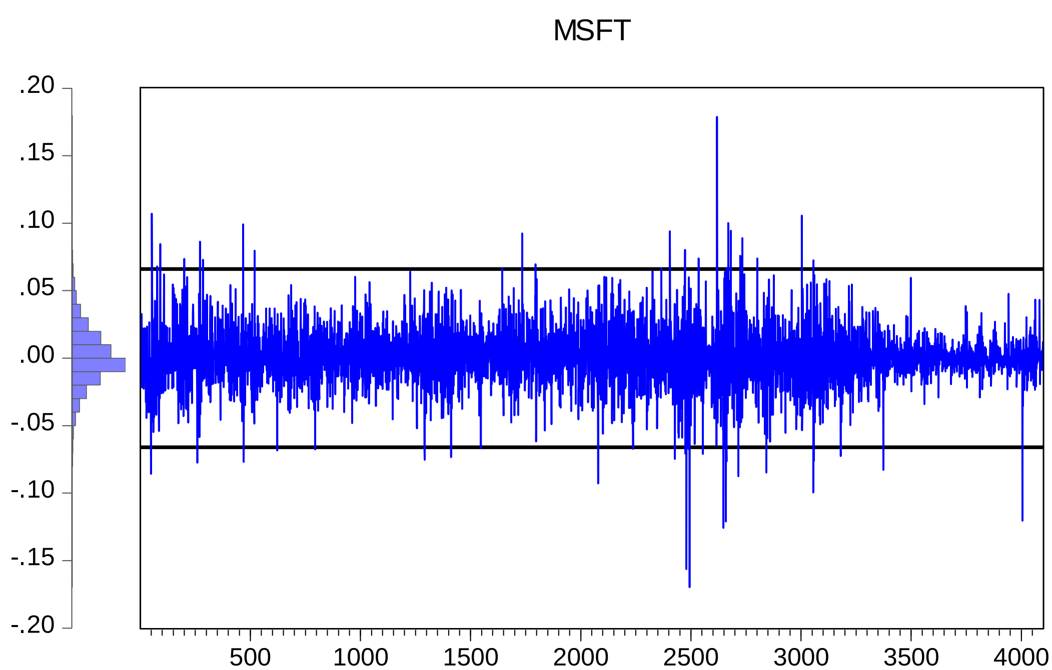
- **Calender effects**

  Returns of the market índices and individual stocks can vary significantly depending on the day of the week (monday returns measure the result of an investment for 72 hours from Friday's close to Monday's), the day of the month, the month of the year, and the proximity of holidays.

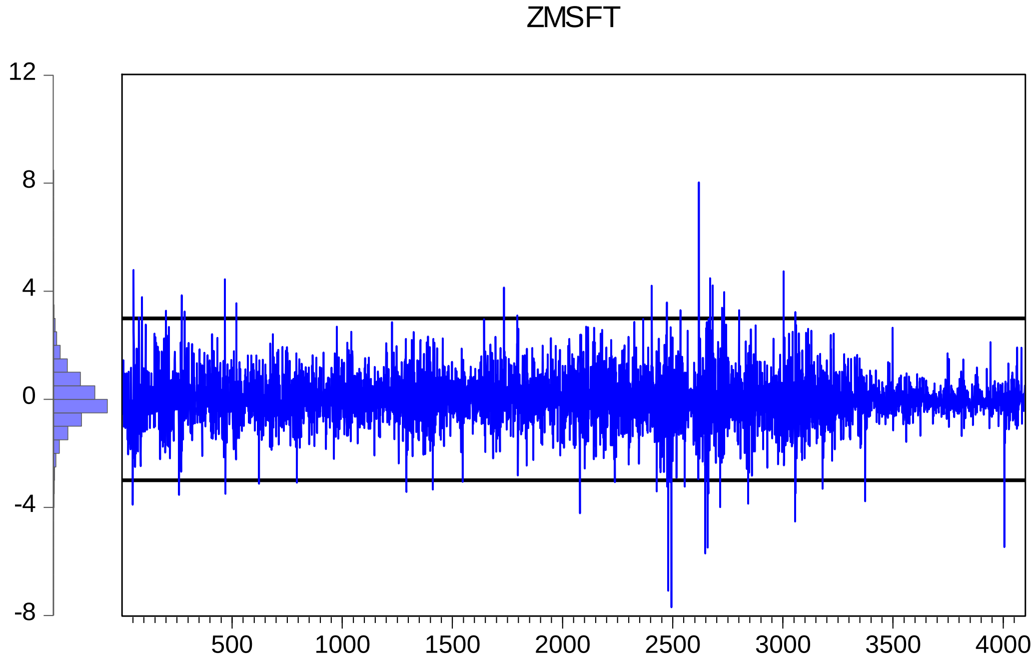
Example: How to interpret kurtosis statistics for log returns of IBM, KO, MSFT and PG stocks? Test normality for MSFT returns?



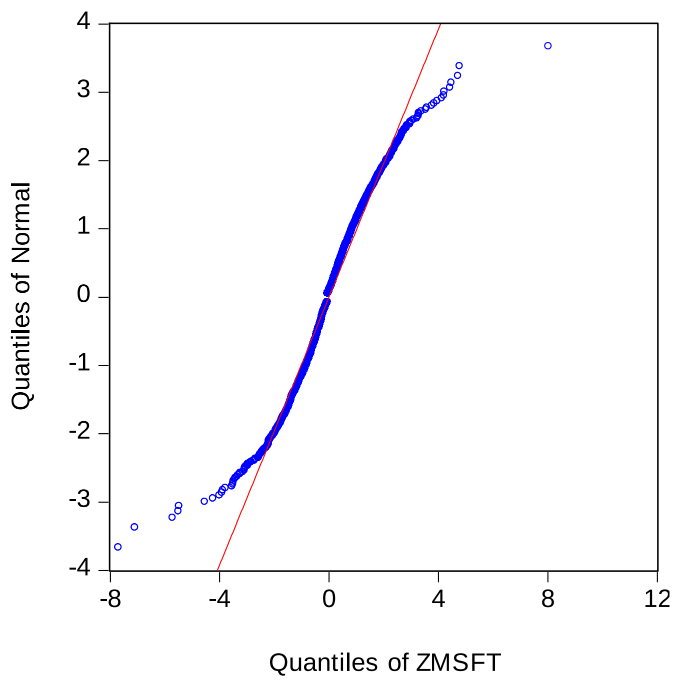
Time plot of daily Microsoft log returns and two standard deviation limits (±2σ)



Time plot of daily Microsoft log returns and three standard deviation limits (±3σ)



Time plot of Microsoft standardized log returns $Z_t = (r_t - \bar{r})/s = (r_t - 0.00082)/0.022159$ and limits ±3



QQ-plot of Microsoft standardized returns

Distributions of returns

Some marginal distributions of asset returns:

- **Normal distribution**
  The simple returns $R_t, t = 1,\ldots,T$ are assumed to be independently and identically distributed as normal. However, we know that the normality assumption is not supported by many empirical asset returns, which tend to have high kurtosis (i.e., fat tails and high peaks).

- **Lognormal distribution**
  It is also assumed that the log returns r_t are independent and identically distributed as normal with fixed mean μ and variance σ^2. The simple returns R_t are then independent and identically distributed as lognormal random variables with mean and variance given by:

$$E(R_t) = \exp\left(\mu + \frac{\sigma^2}{2}\right) - 1 \quad \text{and} \quad Var(R_t) = \exp\left(2\mu + \sigma^2\right)[\exp(\sigma^2) - 1]$$

  Let m_1 e m_2 be the mean and variance of the simple return R_t, then the mean and variance of the log return are r_t given by

$$E(r_t) = \ln\left[\frac{m_1 + 1}{\sqrt{1 + m_2/(1+m_1)^2}}\right] \quad \text{and} \quad Var(r_t) = \ln\left[1 + \frac{m_2}{(1+m_1)^2}\right]$$

  However, the log normal distribution is not consistent with the empirical distribution of an asset return with excess of returns.

- **t-Student distribution**
  A random variable X with t-Student distribution with n degrees of freedom has $E(X) = 0$, $Var(X) = n/(n-2)$, $sk = 0$ and $k = 3 + 6/(n-4)$. When $k > 3$, it follows that $t(n)$ has fat tails. A random variable ε with t-Student distribution with zero mean and unit variance has better statistical properties. So use the transformation $\varepsilon = X/\sqrt{(n-2)/n}$

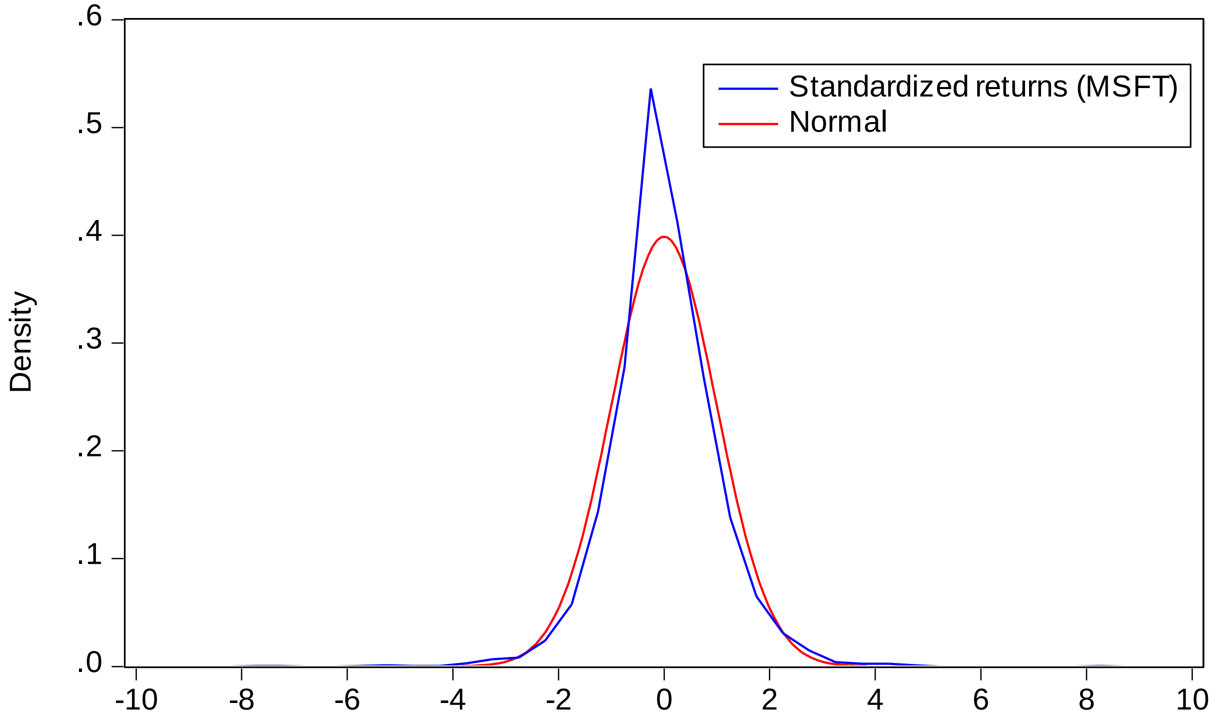
- **Misture of normal distributions**
  Many authors such as Praetz (1972) and Clark (1973) have argued that observed returns come from a misture of normal distributions. An example is

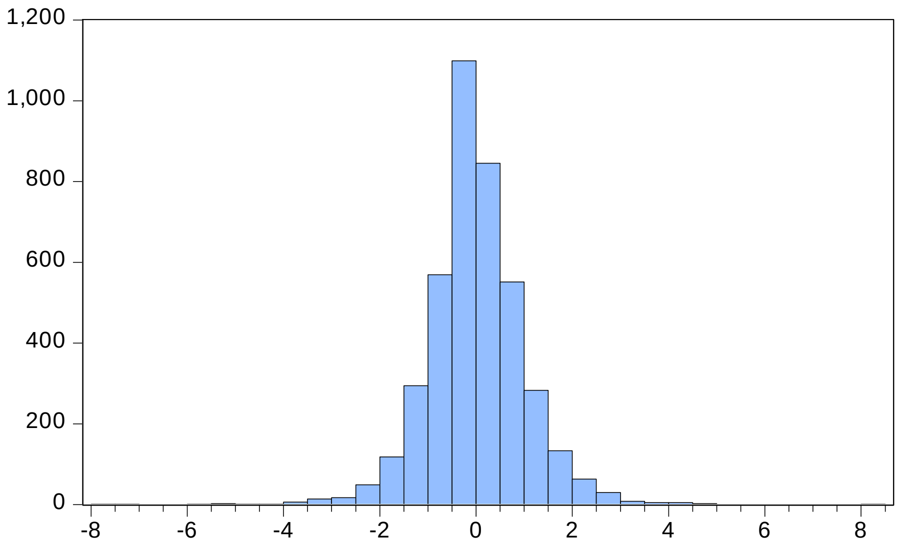
$$r_t \sim (1-X)N(\mu, \sigma_1^2) + XN(\mu, \sigma_2^2),$$

  where X is a Bernoulli random variable such that $P(X=1) = \alpha$ and $P(X=0) = 1-\alpha$ with $0 < \alpha < 1$, σ_1^2 is small and σ_2^2 is relatively large.

Example: The distribution of MSFT standardized log returns

The distribution is not normal: Is is approximately symmetric, it has fat tails and it has a higk peak.



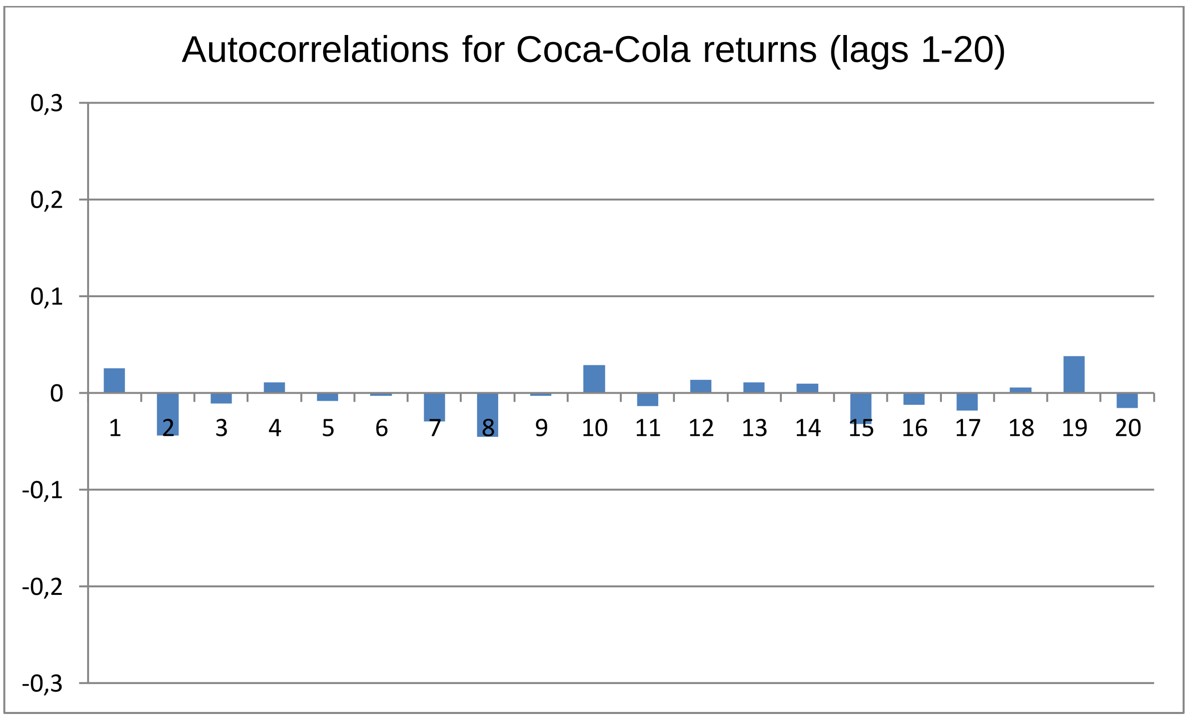


Series: ZMSFT	
Sample 1 4099	
Observations 4099	
Mean	0.000898
Median	-0.036103
Maximum	8.033825
Minimum	-7.697508
Std. Dev.	0.999979
Skewness	-0.041307
Kurtosis	7.471314
Jarque-Bera	3415.743
Probability	0.000000

Autocorrelations of returns

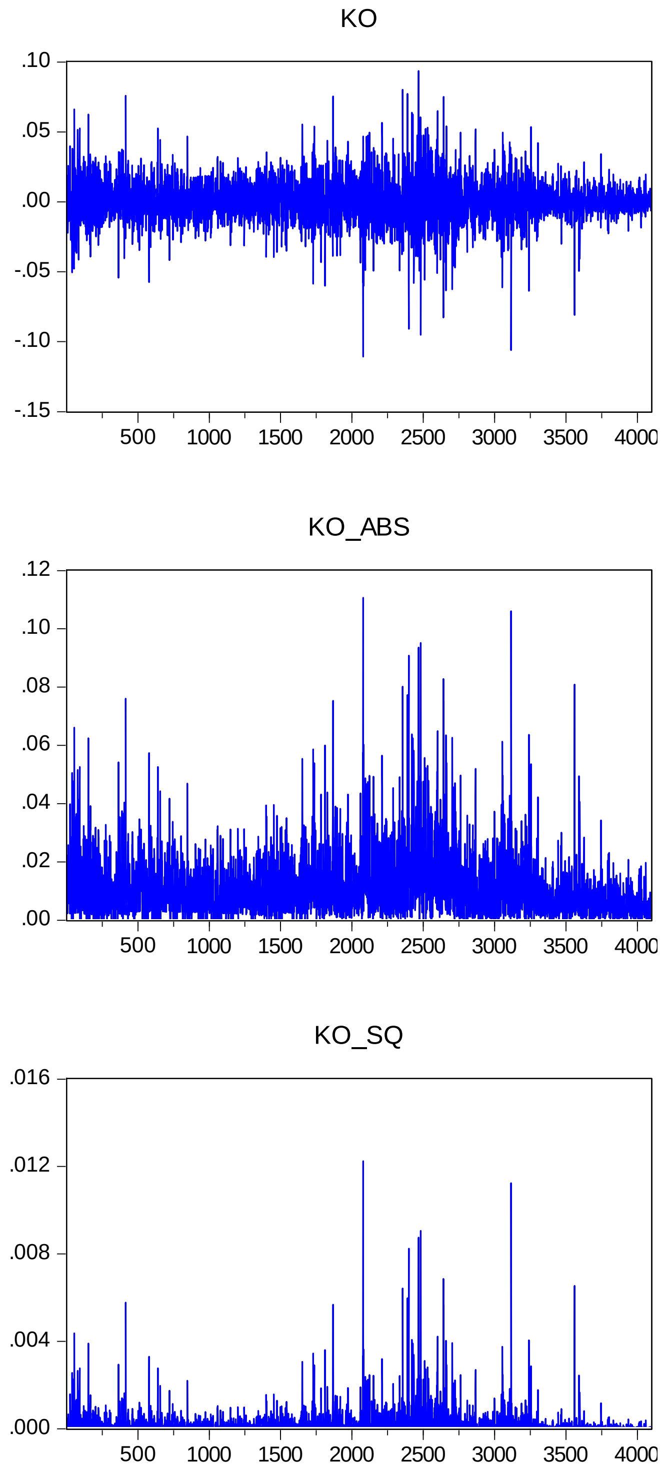
The sample autocorrelations of returns are generally close to zero for different lags. This is one of the most important stylized facts for asset returns. The sample autocorrelation function of returns at lag *k* is given by

$$\hat{\rho}_k = \frac{\sum_{t=1}^{n-k}(r_t - \bar{r})(r_{t+k} - \bar{r})}{\sum_{t=1}^{n}(r_t - \bar{r})^2}, \quad k = 1, 2, \ldots, n-1$$

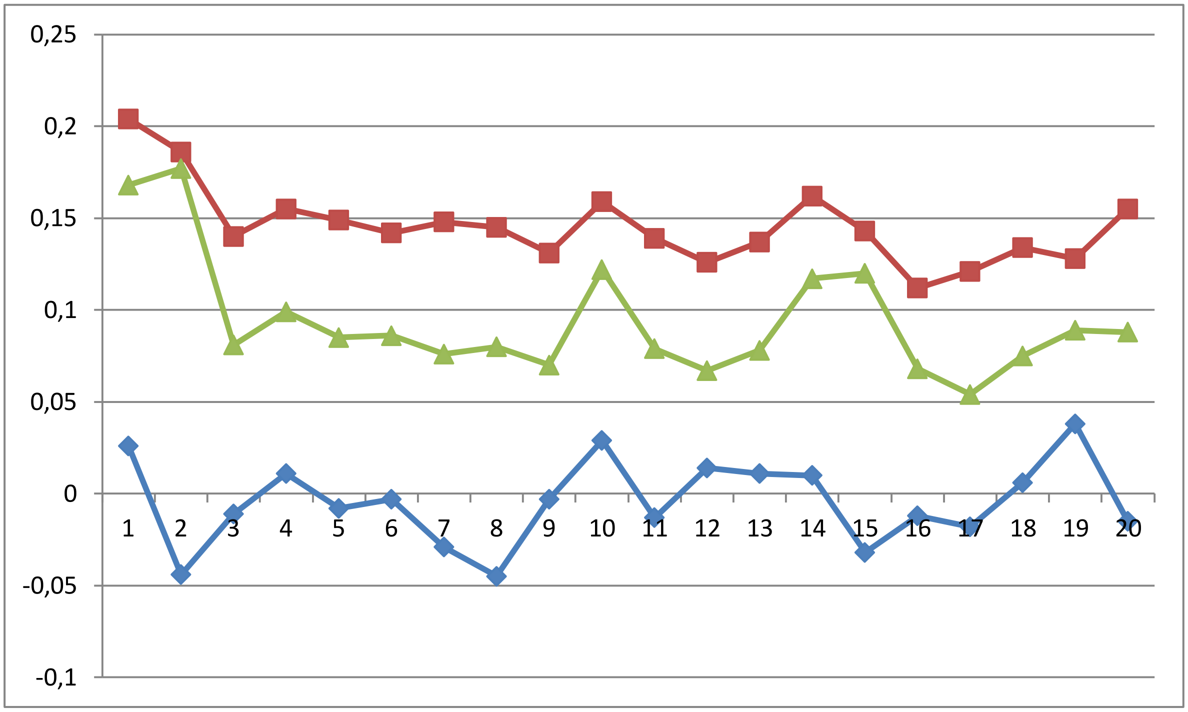


Autocorrelations of transformed returns

There is a positive dependence between absolute returns or squared returns on nearby time lags.



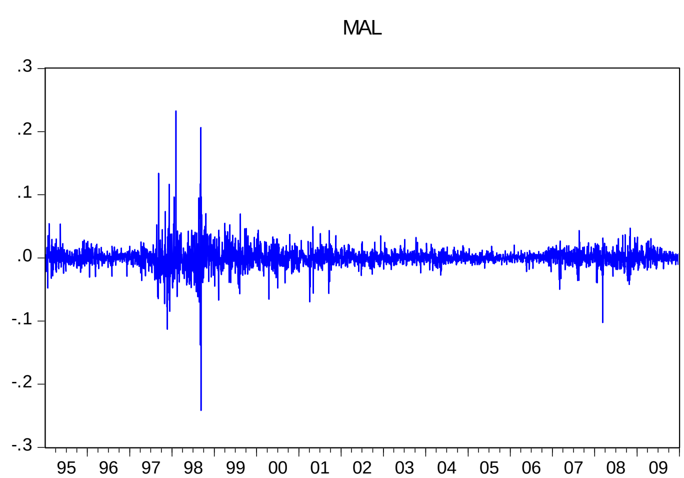
Time plots of Coca-Cola (KO) returns, absolute returns and squared returns

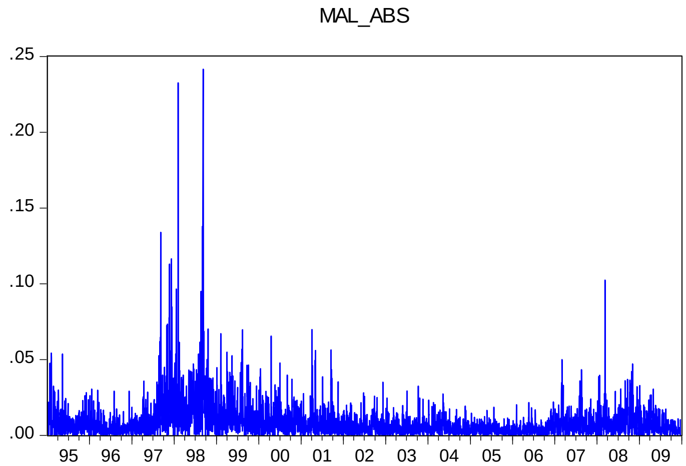


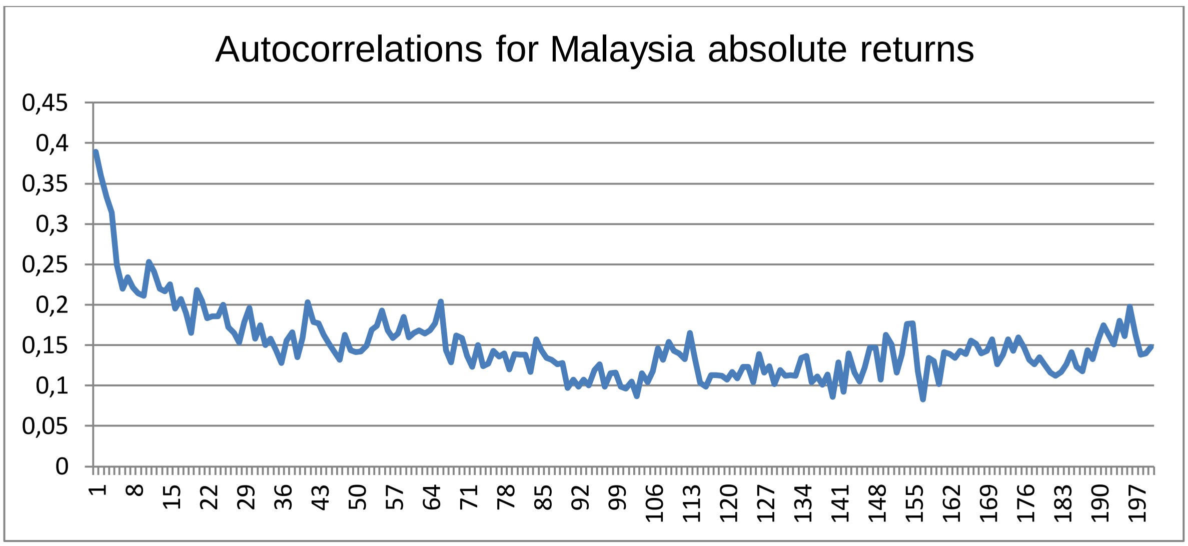
Sample autocorrelations (lags 1-20) of Coca-Cola (KO) returns (blue line), absolute returns (red line) and squared returns (green line)

"*Volatility clustering*"

The phenomenon of "volatility clustering" means that "large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes" (Mandelbrot, 1963). While returns themselves are uncorrelated, absolute returns or their squares display a positive, significant and slowly decaying autocorrelation function. Asset volatility tends to revert to some mean rather than remaining constant or moving in monotonic fashion over time.







Asymmetric effects

The asymmetric effect occurs when returns decline and volatility increases.

Comovements across international stock markets

Return co-movements and volatility co-movoments across international stock markets

Example:

	POR	UK	US	JAP	GER
POR	1.000000	0.596489	0.319881	0.274125	0.584191
UK	0.596489	1.000000	0.475695	0.289925	0.766151
US	0.319881	0.475695	1.000000	0.105774	0.527433
JAP	0.274125	0.289925	0.105774	1.000000	0.255264
GER	0.584191	0.766151	0.527433	0.255264	1.000000

Correlations between international stock markets (returns)

	POR_SQ	UK_SQ	US_SQ	JAP_SQ	GER_SQ
POR_SQ	1.000000	0.613843	0.334370	0.288741	0.505719
UK_SQ	0.613843	1.000000	0.461219	0.318544	0.682555
US_SQ	0.334370	0.461219	1.000000	0.168537	0.556409
JAP_SQ	0.288741	0.318544	0.168537	1.000000	0.215199
GER_SQ	0.505719	0.682555	0.556409	0.215199	1.000000

Correlations between international stock markets (squared returns or unconditional volatilities)

Exercise: Analyze the empirical properties of AIG and CAT returns.